COBORDISM AND GROUP ACTIONS ON MANIFOLDS

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Lecture 1. Bordism and cobordism: basics

Cobordism theory is one of the deepest and most influential parts of algebraic topology, which experienced a spectacular development in the 1960s. Here we summarise the required facts on bordism and cobordism, with the most attention given to complex (co)bordism.

Lecture 2. Formal Group Laws and Hirzebruch Genera

The theory of formal groups originally appeared in algebraic geometry and plays an important role in number theory. *Formal groups laws* were brought into cobordism theory by Mishchenko and Novikov in 1967. They constructed and studied the 'formal group law of geometric cobordisms', otherwise known as the formal group law in complex cobordism. Early applications concerned finite group actions on manifolds. An important result of Quillen followed, asserting that the formal group law in complex cobordism coincides with the universal formal group law, introduced by Lazard in 1954. Subsequent developments included constructions of complex oriented cohomology theories, such as Morava K-theories and elliptic cohomology, and applications to Hirzebruch genera, one of the most important classes of invariants of manifolds.

Lecture 3. Equivariant bordism: geometric and homotopical approach.

Here we review different constructions of *equivariant genera* for stably complex manifolds equipped with compatible actions of a torus T^k. These constructions focus on the notion of the *universal toric genus* $\varPhi$, defined on stably complex T^k-manifolds and taking values in the complex cobordism ring $U^*(BT^k)$ of the classifying space. The universal toric genus $\varPhi$ is defined via the universal transformation between the geometric, homotopic and Borel versions of equivariant cobordism, and goes back to the works of tom Dieck, Krichever and Löffler from the 1970s. The genus $\varPhi$ is an equivariant analogue of the universal Hirzebruch genus corresponding to the identity homomorphism from the complex cobordism ring $\varOmega_U$ to itself.

Lecture 4. Localisation techniques in cobordism

Lecture 5. Rigidity and fibre multiplicativity of genera

Historically, equivariant extensions of genera were first considered by Atiyah and Hirzebruch, who established in 1970 the *rigidity property* of the χ_y-genus of S^1-manifolds. The origins of these concepts lie in the Atiyah–Bott fixed point formula, which also acted as a catalyst for the development of equivariant index theory. This development culminated in the celebrated result of Bott and Taubes establishing the rigidity of the Ochanine–Witten elliptic genus on spin S^1-manifolds.

Here we develop an approach to equivariant genera and rigidity based solely on complex cobordism theory. It allows us to define an equivariant extension and the appropriate concept of rigidity for an arbitrary Hirzebruch genus, and agrees with the classical index-theoretical approach when the genus is the index of an elliptic complex.

Localisation theorems in equivariant generalised cohomology theories appear in the works of tom Dieck, Quillen, Krichever, Kawakubo and elsewhere. We proceed by deducing a localisation formula for the universal toric genus $\varPhi(M)$ in terms of fixed point data. We give several illustrative examples, and describe the consequences for certain non-equivariant genera and their T^k-equivariant extensions.

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