

**ARITHMETIC AND ZARISKI-DENSE SUBGROUPS: WEAK  
COMMENSURABILITY, EIGENVALUE RIGIDITY AND APPLICATIONS TO  
LOCALLY SYMMETRIC SPACES**

A. RAPINCHUK

**Lecture 1** *Eigenvalue rigidity and isospectral and length-commensurable locally symmetric spaces*

- Classical vs. eigenvalue rigidity
- Isospectrality: Can one hear the shape of a drum?
- Length-commensurable spaces

**Lecture 2** *Arithmetic and Zariski-dense subgroups and their weak commensurability*

- A brief overview of algebraic groups and their arithmetic subgroups
- Weak commensurability
- Classification of weakly commensurable arithmetic groups
- Geometric applications

**Lecture 3** *Some techniques: generic elements*

- Existence of generic elements in Zariski-dense subgroups
- Some open problems

**Lecture 4** *Arbitrary Zariski-dense subgroups*

- Back to the general form of eigenvalue rigidity
- Algebraic aspects: quaternion algebras with the same maximal subfields etc.
- Some results and open questions