Kairvaire Problems in Stable Homotopy Theory

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1 Kervaire Problem

Let

$$f : M^{n-1} \looparrowright \mathbb{R}^n$$

be an immersion of a closed, generally speaking, non-orientable manifold in the codimension 1, assumed $n \equiv 2 \pmod 4$. Consider the self-intersection manifold of f, denote this manifold by N^{n-2}.

Consider the characteristic class

$$w_2 \in H^2(N^{n-2}; \mathbb{Z}/2),$$

denote the characteristic number

$$< w_2^{\frac{n-2}{2}}; [N^{n-2}] > \in \mathbb{Z}/2$$

by $\Theta(f)$. The most interesting case is $n = 2^l - 2$, $l \in \mathbb{N}, l \geq 2$.

Problem 1. *Kervaire Problem*

Assume that $n = 2^l - 2$, $l \geq 2$. *For which* l *the following equation*

$$\Theta(f) = 1$$

for an immersion $f : M^{n-1} \looparrowright \mathbb{R}^n$ *is possible?*

2 Generalized Kervaire Problem

Let

$$g : M^{n-k} \looparrowright \mathbb{R}^n$$

be an immersion of a closed, generally speaking, non-oriented manifold, $\dim(M) = n-k$, where $n \equiv 2 \pmod 4$, $n > k$. Denote by $\kappa : M^k \looparrowright \mathbb{RP}^\infty$ a prescribed characteristic class.

Assume that the following isomorphism:

$$\Psi : \nu(g) \cong k\kappa,$$

which is called a skew-framing of the immersion g, is well-defined; where $\nu(g)$ is the normal bundle of g, $k\kappa$ is the Whitney sum of k copies of the line bundle κ (more precisely, $\Psi : \nu(g) \cong \kappa^*(\gamma)$, where γ is the only non-trivial bundle over $\mathbb{RP}^\infty$.)

The triple

$$(g, \kappa, \Psi)$$

represents an element in the cobordism group

$$Imm^{sf}(n-k, k).$$

Assume that $k = \frac{n}{2}$, define the characteristic number

$$\theta(g, \kappa, \Psi) \in \mathbb{Z}/2$$

as the parity of self-intersection points of the immersion g (we assume that g is self-transversal). In fact, θ depends not of (κ, Ψ) and $\theta(g, \kappa, \Psi) = \theta(g)$.

For a given pair (k_-, k_+), which satisfies the inequality:

$$n > k_+ > \frac{n}{2} > k_- \geq 0,$$

the following homomorphisms are well-defined:

$$Imm^{sf}(k_+, n-k_+) \stackrel{J^{k_+}_{\frac{n}{2}}}{\longrightarrow} Imm^{sf}(\frac{n}{2}, \frac{n}{2}),$$

$$Imm^{sf}(\frac{n}{2}, \frac{n}{2}) \stackrel{J^{\frac{n}{2}}_{k_-}}{\longrightarrow} Imm^{sf}(k_-, n-k_-).$$

Definition 1. *Denote the subgroup*

$$Im(J_{\frac{n}{2}}^{k_+}) \cap Ker(J_{k_-}^{\frac{n}{2}}) \subset Imm^{sf}(\frac{n}{2}, \frac{n}{2})$$

by

$$I_{k_+,k_-}^{n} \subset Imm^{sf}(\frac{n}{2}, \frac{n}{2}).$$

The 2-parameter family of subgroups determines the double filtration of the group $Imm^{sf}(\frac{n}{2}, \frac{n}{2})$.

Problem 2. *Generalized Kervaire Problem*

For which (k_+, k_-) *the restriction of the homomorphism* θ *on the subgroup* $I^n_{k_+,k_-}$ *is non-trivial?*

(there exists

$$[(g,\kappa,\Psi)] \in I^n_{k_+,k_-} \subset Imm^{sf}(\frac{n}{2},\frac{n}{2}),$$

for which $\theta(g) = 1$*?)*

Proposition 1.

Generalized Kervaire Problem for the term $I^n_{\frac{n}{2},\frac{n}{2}-1}$ *is positively solved, iff* $n = 2, 6, 14$.

Proof of Proposition 1

The group $I^n_{\frac{n}{2},\frac{n}{2}-1}$ is represented by triples (g,Ξ), where $g : M_1^{\frac{n}{2}} \looparrowright \mathbb{R}^n$, Ξ–is the trivialization of the normal bundle of g: $\Xi : \nu_g \cong \frac{n}{2}\varepsilon$. Therefore, (g,Ξ) represents an element in $\Pi_{\frac{n}{2}}$.

The value $\theta(g,\Xi)$ coincides with the stable Hopf invariant $h([g,\Xi])$. By the Adams Theorem $h([g,\Xi]) = 0$, if $l \geq 4$; the homomorphism $h : \Pi_{\frac{n}{2}} \to \mathbb{Z}/2$ is onto, iff $n = 2, 6, 14$. $\square$

3 Strong Kervaire Problem

Define the element

$$w_k \in \pi_{2k-1}(S^k)$$

by the Whitehead square of the generator $i_k \in \pi_k(S^k)$, $k \equiv 1 \pmod 2$.

Problem 3. *Strong Kervaire Problem*

Assume $k = 2^l - 1$. For which l the element w_k is halved?

Proposition 2. *Exercises*

Strong Kervaire Problem is positively solved, $k = 2^l - 1$*, iff Generalized Kervaire Problem for the term* $I^n_{\frac{n}{2},\frac{n}{2}-2}$*,* $n = 2k$*, is positively solved.*

Assume that Kervaire Problem is positively solved $n = 2^{l+1} - 2$*, then Generalized Kervaire Problem is positively solved for the term* $I^n_{n-1,0}$*.*

4 Barratt-Jones-Mahowald Theorem [1983]

Theorem 1.

Assume that for a given $n = 2^{l+1} - 2$ *Strong Kervaire Problem is positively solved (this means that the homomorphism*

$$\theta : I^{n}_{\frac{n}{2},\frac{n}{2}-2} \to \mathbb{Z}/2$$

is onto). Then Kervaire Problem for $n' = 2^{l} - 2$ *is positively solved (this means that the homomorphism*

$$\Theta : Imm^{sf}(n'-1,1) \to \mathbb{Z}/2$$

is onto).

Denote by $V_{k,2}$ the Stiefel manifold $V_{k,2}$ of 2-framed in $\mathbb{R}^k$; the most interesting case is $k = 2^l - 1$, $l \geq 2$. Denote by $* : V_{k,2} \to V_{k,2}$ the involution by the formula: $\{e_1, e_2\} \mapsto \{e_1, -e_2\}$.

Definition 2. *Let us say that the manifold $V_{k,2}$ is neutral, if the involution $*$ is homotopic to the identity.*

Theorem 2. *James Theorem [1976]*

The manifold $V_{k,2}$*,* $k = 2^l - 1$*,* $l \geq 2$ *is neutral, iff the element* $w_k \in \pi_{2k-1}(S^k)$ *is halved. (Strong Kervaire Problem is positively solved in* $I^n_{\frac{n}{2},\frac{n}{2}-2}$*,* $n = 2k$*).*

Example 1. *The manifold $V_{k,2}$ is neutral for $k = 7$.*

Proof of Example 1

Define a homotopy $F(t) : V_{7,2} \to V_{7,2}$, $F(0) = Id$, $F(1) = I$. Consider the standard inclusion $\mathbb{R}^7 \subset \mathbb{R}^8$, which is orthogonal to the base vector $\mathbf{1} \in \mathbb{R}^8$. Let $\mathbf{f}_1 \in \mathbb{R}^7 \subset \mathbb{R}^8$ be a base vector, which is orthogonal to the vector $1 \neq \mathbf{f}_1$. Denote by $L(\mathbf{f}_1)$ the orthogonal complement to the vector $\mathbf{f}_1$ in the subspace $\mathbb{R}^7$. By construction, $\mathbf{e}_2 \in (\mathbf{e}_1)^{\perp}$.

A one-parameter family of orthogonal transformation $G(t, \mathbf{e}_1) : \mathbb{R}^7 \to \mathbb{R}^7$ which transforms the subspace $(\mathbf{e}_1)^{\perp}$ to itself, is well-defined by multiplication of vectors from $(\mathbf{e}_1)^{\perp}$ by the unite $\mathbf{1}\cos(t\pi) + \mathbf{e}_1 \sin(t\pi)$.

Denote by $F(t, \mathbf{e}_1) : \mathbb{R}^7 \to \mathbb{R}^7$ the transformation, which is the identity in the subspace generated by the vector $\mathbf{e}_1$ and which coincides to $G(t, \mathbf{e}_1)$ on the subspace $(\mathbf{e}_1)^{\perp}$. The homotopy $F(t, \mathbf{e}_1)$ is required.

Remark

Example 1 could be interesting with respect to results [Cantarella at all (2012)]. The space $V_{k,2}$ is the model of random closed k-gones on the plane with the total unite length of edges. The homotopy $F(t)$ determines the transformation of the space of k-gones to itself, a gone is transformed to its mirror image.

Kervaire Problem is positively solved for $n = 2, 6, 14, 30, 62$. In the case $n \geq 254$ Kervaire Problem is negatively solved by Hill-Hopkins-Ravenel Theorem [2010].

The following conjecture is open.

Conjecture 1. *Snaith Conjecture [2009]*

Kervaire Problem is negatively solved for $n = 126$.

5 Solution of Strong Kervaire Problem for $\frac{n}{2} = 15$

Define a closed manifold M^{15} as a semi-direct product of $S^7 \times S^7$ and the circle S^1, which is fibered over S^1 with the fiber $S^7 \times S^7$. The self-identification of the fiber $S^7 \times S^7$ along S^1 is given by the involution $S^7 \times S^7 \to S^7 \times S^7$, which permutes the factors. The orientation class $w_1(M^{15}) = \kappa \in H^1(M^{15}; \mathbb{Z}/2)$ is the pull-back of the generator $H^1(S^1)$ by the projection $M^{15} \to S^1$.

Let us define an immersion $\varphi : M^{15} \looparrowright \mathbb{R}^{30}$, which is skew-framed by $\Psi : \nu_\varphi \cong 15\kappa$. Consider the manifold $\hat{K}^{15} = \mathbb{RP}^7 \times \mathbb{RP}^7 \tilde{\times} S^1$, which is fibered over S^1 with the fiber $\mathbb{RP}^7 \times \mathbb{RP}^7$. The self-identification of the fiber $\mathbb{RP}^7 \times \mathbb{RP}^7$ over a marked point $pt \in S^1$ is given by the involution $\mathbb{RP}^7 \times \mathbb{RP}^7 \to \mathbb{RP}^7 \times \mathbb{RP}^7$, which permutes the factors.

An immersion $\hat{\varphi} : \hat{K}^{15} \looparrowright \mathbb{R}^{30}$ is well-defined, this immersion is skew-framed by $\hat{\Psi} : \nu_{\hat{\varphi}} \cong 15\hat{\kappa}$. Consider the standard 4-sheeted covering $p : M^{15} = (S^7 \times S^7)\tilde{\times} S^1 \to \hat{K}^{15}$, which is induced by the coordinate coverings $S^7 \to \mathbb{R}\mathrm{P}^7$. The immersion φ is well-defined by the formula: $\varphi = \hat{\varphi} \circ p$.

A skew-framing Ψ is well-defined by the formula: $\Psi = \varphi^*(\hat{\Psi})$. The value $\theta(\varphi, \kappa, \Psi)$ is calculated as a parity of self-intersection points of the immersion $\varphi = \hat{\varphi} \circ p$, which is generically deformed into $\mathbb{R}^{30}$ by a small perturbation. The formula

$$\theta(\varphi, \kappa, \Psi) = 1$$

is satisfied.

6 The Jones Manifold

In the paper [J.D.S. Jones (1978)] a framed closed manifold $\tilde{M}^{30}$ of dimension 30 with Arf-invariant 1 is constructed.

Definition of the quadratic form $q : H_{2k+1}(\tilde{M}^{4k+2}) \to \mathbb{Z}/2$ for a framed $4k+2$-manifold

Let $(\tilde{M}^{4k+2}, \Xi)$ be an arbitrary closed manifold with a prescribed trivialization of the stable normal bundle $\Xi : \nu_{\tilde{M}} \cong r\varepsilon$, $r-2 \geq 4k+2 = dim(\tilde{M})$. The framing Ξ determines up to regular concordance a 0-codimension immersion $\varphi : (\tilde{M}^{4k+2} \setminus pt) \looparrowright \mathbb{R}^{4k+2}$ of the open manifold.

Let $x \in H_{2k+1}(\tilde{M}^{4k+2};\mathbb{Z}/2)$ be a homology class. By Thom Theorem and Whitney Theorem the class x is represented by an embedding $l : K^{2k+1} \subset \tilde{M}^{4k+2}$, where K^{2k+1} is a closed, generally speaking, non-oriented $(2k+1)$-manifold.

Consider the immersion $\varphi \circ l : K^{2k+1} \subset (\tilde{M}^{4k+2} \setminus pt) \looparrowright \mathbb{R}^{4k+2}$, assume that this immersion is generic. Define $q(x, l)$ as the parity of self-intersection points of the immersion $\varphi \circ l$.

The value $q : H \to \mathbb{Z}/2$, $H = H_{2k+1}(\tilde{M}^{4k+2};\mathbb{Z}/2)$, depends only on x and depends not on the cycle $l : K^{2k+1} \subset \tilde{M}^{4k+2}$, which represents $x \in H$. The form q is quadratic non-degenerated, this form is associated with the cap-product of $(2k+1)$-cycles on $\tilde{M}^{4k+2}$.

Let

$$\{\mathbf{e}_1, \mathbf{f}_1; \dots \mathbf{e}_s, \mathbf{f}_s\}$$

be the Hamiltonian base of H with respect to the cap-product of cycles.

Define the Arf-invariant of the form q by the formula:

$$Arf(q) = \sum_{i=1}^{s} q(\mathbf{e}_i) q(\mathbf{f}_i) \in \mathbb{Z}/2.$$

$Arf(q)$ is invariant with respect to framed surgery of the framed manifold $(\tilde{M}^{4k+2}, \Xi)$. By Pontryagin Theorem, $Arf(q)$ determines a homomorphism $\Pi_{30} \to \mathbb{Z}/2$.

Let us construct a stably parallelized manifold $\tilde{M}^{30}$, which is called Jones Manifold. Denote by P^2 the closed non-orientable surface of the Euler characteristic +3 (the connected sum of the projective plane and the torus). Consider an arbitrary embedding $i_P : P^2 \subset \mathbb{R}^{35}$, denote by ν_P the normal bundle of the embedding i_P, $\dim(\nu_P) = 33$.

Denote by $\nu_0 \subset \nu_P$ a trivial subbundle of the dimension 32, $\nu_0 \cong 32\varepsilon$. Let us write $\nu_P \cong \hat{\nu}_P \oplus \nu_0$, where $\hat{\nu}_P$ is the line orienting normal bundle over P. Decompose the fiber of ν_0 over a marked point $pt \in P^2$ into 4 blocks, denote the blocks by A, B, C, D, the each block is a 8-dimensional vector space.

Consider the decomposition of the surface $P^2 \cong Q^2 \cup_S R^2$, where Q^2 is the projective plane with hole, R^2 is the torus with hole, the two surfaces Q^2, R^2 are glued along the common boundary circle S. Denote by $a \in H_1(Q^2)$ the generator on the projective plane, by $b_1, b_2 \in H_1(P^2)$ generators on the torus.

Define a flat 4-bundle $\mu : E(\mu) \to P$, the structure group of this bundle is given by the permutation of the 1 and 3 base vectors along the loop on Q^2, represented a; by the permutation $(1,2)(3,4)$ of the base vectors along the loop, represented b_1; and by the permutation $(2,3)(4,1)$ of the base vectors along the loop, represented b_2.

The bundle μ admits the dihedral structure group $\mathbf{D}$, this group contains 8 elements, the formulas determine a representation $\mu : \pi_1(P^2) \to O(4)$.

Consider the bundle 8μ, the Whitney sum of 8 copies of the bundle μ. The bundle 8μ over P^2 is trivial, and an isomorphism $8\mu \cong \nu_0$ is well-defined.

The fiber of the bundle 8μ over the marked point $pt \in P$ is decomposed into 8-bundles as follows: $A \oplus B \oplus C \oplus D$. Obviously, $\mu \cong \varepsilon \oplus \hat{\mu}$, where $\dim(\hat{\mu}) = 3$. It is not hard to prove, that $w_1(\hat{\mu}) \in H^1(P^2)$ is dual to a, $w_2(\hat{\mu}) \in H^2(P^2)$ represents the fundamental class on P^2. We have $\hat{\Xi} : \hat{\nu}_P \oplus \hat{\mu} \cong 4\varepsilon$ and $\Xi : \hat{\nu}_P \oplus \mu \cong 5\varepsilon$.

Consider the fiber $A \oplus B \oplus C \oplus D$ of ν_0 over the marked point $pt \in P^2$ and consider the Hopf immersion $h : S^7 \looparrowright \mathbb{R}^8$, which determines a generator of $[\Pi_7]_{(2)} \cong \mathbb{Z}/16$. Consider the Cartesian product

$$h^4 : S^7 \looparrowright A \oplus B \oplus C \oplus D$$

of 4 copies of the immersion h.

Define the required manifold $\tilde{M}^{30}$ as the semi-direct product $P^2 \tilde{\times} (S^7)^4$, where the dihedral group $\mathbf{D}$ transforms the fiber $(S^7)^4$, keeping locally the Cartesian decomposition. By the construction, $\mathbf{D}$ keeps the immersion h^4 and keeps the framing of this immersion.

The immersion $\tilde{M}^{30} \looparrowright E(8\mu)$ into the total space of the bundle 8μ is well-defined. Because $E(8\mu)$ is diffeomorphic to the total space of the bundle ν_0, the immersion $f : \tilde{M}^{30} \looparrowright \mathbb{R}^{35}$ is well-defined. The normal bundle ν_f of the immersion f is isomorphic to the Whitney sum $\hat{\nu}_P \oplus \mu$; the normal bundle ν_f is trivial. The manifold $\tilde{M}^{30}$ admits a framing Ξ, the pair $(\tilde{M}^{30}, \Xi)$ determines an element in Π_{30}. Jones Manifold is well-defined.

Hamiltonian base in the middle homology of the Jones Manifold

The vector space $H_{15}(\tilde{M}^{30})$ is 8-dimensional. For Hamiltonian cycles $\{\mathbf{e}_i, \mathbf{f}_i\}$, $i = 1, 2, 3$ we get $q(\mathbf{e}_i) = q(\mathbf{f}_i) = 1$. For the Hamiltonian cycles $\{\mathbf{e}_4, \mathbf{f}_4\}$ we have $q(\mathbf{e}_4) = q(\mathbf{f}_4) = 0$. As the result we have

$$Arf(\tilde{M}^{30}, \Xi) = 1.$$

7 Browder Theorem, Eccles Theorem, the positive solution of Kervaire Problem in dimension $n = 30$

An element α in the stable homotopy group of spheres

$$\Pi_{30} = \pi_{n+30}(S^n), \quad n \geq 32,$$

is represented by a framed manifold (M^{30}, Ξ).

By Browder Theorem [1969], a framed manifold (M^{30}, Ξ) has Arf-invariant one iff the element α in Π_{30} is detected by a secondary cohomological operation, which is based on the Adem relation in the Steenrod algebra:

$$Sq^{2^j} Sq^{2^j} + \sum_{i=1}^{j-1} Sq^{2^{j+1}-2^i} Sq^{2^i} = 0,$$

where $j = 4$.

By Eccles Theorem [1981], the element α is detected by the considered cohomology operation, iff the element $\beta = \lambda_*(\alpha) \in \pi_{29+n}(\Sigma^n(K(\mathbb{Z}/2,1)))$, which is associated with α by the Khan-Priddy mapping $\lambda : Q\mathbb{RP}^\infty \to Q(S^0)$, is detected by the operation Sq^{16} in the cone of the mapping β.

Pontyagin-Thom-Wells Construction determines an element β by an immersion $f : M^{29} \looparrowright \mathbb{R}^{30}$ in the codimension 1. Moreover, the characteristic number $\Theta(f) = 1$, iff Sq^{16} detects β.

Because Jones Manifold is a framed manifold with the Arf-invariant one, there exists an immersion $F : M^{29} \looparrowright \mathbb{R}^{30}$ with $\Theta(f) = 1$. Kervaire Problem is positively solved in dimension $n = 30$.

8 Eccles-Wood filtration for a standard Kervaire 30-manifold

Kervaire [1960] constructed a closed 4-connected PL-manifold of dimension 10, which admits no smooth structure.

A standard Kervaire 30-manifold is defined as a result of a normal surgery of the Jones manifold $\tilde{M}^{30}$ into a 14-connected manifold $\tilde{N}^{30}$ with

$$dim(H_{15}(\tilde{N}^{30}; \mathbb{Z}/2)) = 2.$$

A standard Kervaire manifold is well-defined up to a connected sum with a homotopy 30-sphere. By construction, $\tilde{N}^{30}$ is framed.

The punctured standard Kervaire manifold is unique, in S^{31} it is defined as a fiber over the trefoil knot:

$$z_1^2 + \cdots + z_{15}^2 + z_{16}^3 = 0.$$

In particular, $\tilde{N}^{30}$ admits a PL-embedding into $\mathbb{R}^{32}$.

Proposition 3. *There are no smooth embedding* $\tilde{N}^{30} \times D^{14} \subset \mathbb{R}^{46}$*, where* $\tilde{N}^{30}$ *is a standard Kervaire manifold,* D^{14} *is the standard disk.*

To prove the statement we recall the Eccles-Wood filtration [1979] for framed manifolds.

Let N^n be a closed framed manifold (let us restrict to the case $n \equiv 0 \pmod 2$), with a prescribed trivialization of the stable normal bundle: $\Xi : \nu_N \cong r\varepsilon$ of a dimension r, $r \geq n+2$.

One say that (N^n, Ψ) admits , at least, the Eccles-Wood filtration (k_+, k_-),

$$n > k_+ > \frac{n}{2} \ge k_- \ge 0,$$

if there exists an embedding $N^n \subset \mathbb{R}^{2n-k_-}$ with the normal vector field ψ of the dimension $k_+ - k_-$ (of the codimension $n - k_+$).

A positive solution of Generalized Kervaire Problem for $I^n_{k_+,k_-}$ gives an obstruction to find an embedding in the regular cobordism class of an immersion $N^n \looparrowright \mathbb{R}^{2n-k_-}$, framed in codimension $n-k_+$.

Proposition 4. *A standard Kervaire manifold* $(\tilde{N}^{30}, \Xi)$ *admits not the Eccles-Wood filtration* $(28, 14)$.

Theorem 3. *(unpublished)*

A standard Kervaire manifold $(\tilde{N}^{30}, \Xi)$ *admits not the Eccles-Wood filtration* $(23, 14)$ *(by the author, M.Cencelj, and D.Repovš [2010]).*

Proof of Proposition 4

Assume, there exists an embedding $f : \tilde{N}^{30} \subset \mathbb{R}^{46}$, for which the normal bundle ν_f admits 14 linear independent sections. Let us prove that the immersion f cannot be an embedding. By the assumption, an isomorphism of the normal bundle $\nu_f \cong \eta \oplus 14\varepsilon$, where η is a real 2–bundle, is well-defined.

Because $\pi_1(\tilde{N}^{30}) = \pi_2(\tilde{N}^{30}) = 0$, the bundle η is a trivial bundle. A framed immersion (f, Ξ), $\Xi : \nu_f \cong 16\varepsilon$ is well-defined. It is well-known, that the Arf-invariant for a framed manifold $(\tilde{N}^{30}, \Xi) \in \Pi_{30}$ depends not on a framing. Therefore for the framed immersion (f, Ξ) we get $Arf(f, \Xi) = 1$.

Let us consider a hyperprojection $\pi : \mathbb{R}^{46} \to \mathbb{R}^{45}$ and consider the mapping $g = \pi \circ f$, by [C. P. Rourke, B. J. Sanderson, 2001] this mapping is a framed immersion (by a framing $\hat{\Xi}$, where $\hat{\Xi} \oplus Id_{\varepsilon} \cong \Xi$).

Consider the self-intersection manifold of the framed immersion $g : \tilde{N}^{30} \looparrowright \mathbb{R}^{45}$. The element $[(g, \Xi)])$, in the cobordism group $Imm^{sf}(15, 15)$ is well-defined. By construction, $[(g, \Psi)] \in K^{30}_{29,14}$ (a priori we have $[(g, \Psi)] \in K^{30}_{28,14}$). The first index k_1 of the filtration is the maximal, therefore by Browder and Eccles Theorems the equation $\Theta([(g, \Psi)]) = 1$ is satisfied.

But, for a framed immersion g one get: $\theta([(g,\Psi)]) = 0$. Therefore an embedding $\tilde{N}^{30} \times D^{14} \subset \mathbb{R}^{46}$ cannot exists. $\square$

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